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Quaking Aspen Productivity Recovers After Repeated Prescribed Fire

D.A. Perala

In 1974, I reported a study where the productivity of quaking aspen (*Populus tremuloides* Michx.) suffered in the short term from a single intense prescribed fire in slashings left after harvest (Perala 1974a) and from subsequent reburns (Perala 1974b). The purpose of the study was to determine (1) whether a single prescribed fire could provide an alternative means to control residual overstories that hamper growth of regenerating aspen stands, and (2) if repeated burning is practical to maintain brushy wildlife habitat or even to aid conversion to conifers by preparing the site for planting. About midway through a commercial rotation, the study yields data that document continued depression of aspen productivity after one burn, but partial or complete recovery after two burns. My objective in this paper is to evaluate these burning effects on productivity by examining effects on site index, tree size, and stockability.

Productivity is defined as stand yield (standing crop here) with respect to stand age. Most commonly, productivity is indexed to stand height with respect to age, *i.e.*, site index. For aspen, the monomolecular equation of Lundgren and Dolid (1970) exemplifies this relationship:

$$S=H/[1.48*[1-\exp(-0.0214*A)]^{.9377}] \quad [1]$$

where

S=site index, m at age 50

H=total height of dominant aspen, m

A=total stand age

Site index is usually considered fixed for a given tree genotype on a given site; however, because height growth is sensitive to edaphic and climatic inputs, perturbations that affect resource supply (*e.g.*, climate change, soil damage) can alter measured site index.

Standing crop has two mensurational components: tree size and tree stocking. Tree size is related to age and site index (McFadden and Oliver 1988, Perala *et al.* 1994), for example:

$$W=(a*S^c)*A^b \quad [2]$$

where

W=mean tree total above-ground biomass

(a*S^c)=scale parameter for a given site index

b=shape parameter

Population stocking can be evaluated with a variant of the size-density and self-thinning relationships (Reineke 1933, Yoda *et al.* 1963),

$$N=a*W^{-b} \quad [3]$$

where

N=maximum stocking density

This relationship expresses "stockability" (DeBell *et al.* 1989, Harms *et al.* 1994); *i.e.*, variability in maximum stocking density (and therefore standing crop) supported by stands of a given mean tree size. This concept was recently applied successfully to quaking aspen by Perala *et al.* (1994).

A change in size, stocking, or site index will be noticed as a change in yield at a given time. Although all these generally depend on one another, the relationships are not invariable (Perala *et al.* 1994), and will be shown here to differ greatly among treatments and between aspen and its woody associates.

Don Perala is a Principal Silviculturist (retired) from the North Central Forest Experiment Station in Grand Rapids, Minnesota.

The technical aspects of conducting prescribed burns in the aspen type were reviewed by Quintilio *et al.* (1991). Jones and DeByle (1985) and Rouse (1986) most recently reviewed the response of aspen to fire.

METHODS

Experimental

Stands, treatments, data acquisition, and early response are described in detail in Perala (1974a,b). To iterate, a quaking aspen stand (basal area 30 m² ha⁻¹) having a strong sub-merchantable hardwood component (8 m² ha⁻¹) on a good, well-drained site in north central Minnesota, was divided into 12 1-ha treatment plots, commercially logged of aspen in early summer 1965, and burned on the following schedule:

- B0..No burning...all residual trees >2.5 cm d.b.h. felled after logging,
- B1..Single burn, spring 1967, in cured slash fuels,
- B2..As in B1 above plus a repeat burn in spring 1969, and
- B3..As in B1 above plus a repeat burn in autumn 1970.

The study is completely randomized and replicated 3 times.

Before logging, diameters of all trees >2.5 cm d.b.h. were measured on each of four systematically arranged permanent 400-m² circular plots per treatment plot. Total height of three dominant and codominant aspens was also measured on each of these plots with an Abney level. Within each of these plots, four 8-m² permanent circular sample plots were installed to sample slash and the forest floor and to estimate fire energy (Beaufait 1966). These same plots were used to sample woody regeneration annually until 1977 and then on about a 5-year schedule until spring 1990. Stem caliper of shrubs and tree regeneration (d.b.h.<2.5 cm) was recorded by 0.1 inch classes determined with a notched gauge at 1.0 foot height. In 1983, the caliper height was lowered to 15 cm and new gauges having 2-mm caliper classes were used to comply with generally used methods. After 5 or 6 years of growth, trees attaining sapling size (>2.5 cm d.b.h.) were measured at 1.37 m height with a steel tape. In 1990, trees were measured on four 200-m² circular plots

per treatment plot using the original plot centers. Also, total height of the three tallest aspen trees on each of these plots was measured with a clinometer.

Variables

The primary response variable was woody biomass estimated from allometric equations (Perala and Alban 1993) applied to the field data. Before these equations could be applied, the English-unit calipers at 1.0 ft required adjustment to SI units at 15 cm from the equation,

$$\text{Caliper (mm@ 15 cm)} = 0.9744 + 25.733 * \text{Caliper (inches @ 1 foot)} \quad [4]$$

This equation was developed from dual stem measurements taken during 1990 of 123 trees <25 mm d.b.h. comprising 8 species and 102 shrubs of 7 species. These were measured with a notched gauge by 0.1 inch caliper classes at 1.0 foot up the stem and by 2-mm caliper classes at 15 cm height. The SI calipers were regressed over the English calipers giving an adjusted R²=0.988, standard error=1.54 mm, and Student's t=5.2 and 135.6 for constant and coefficient, respectively. Incidentally, the coefficient is not significantly greater (t=1.76, p>0.05) than 25.4 (the conversion of English to SI units), so for all practical purposes, caliper at 15 cm is simply 1 mm larger than at 30 cm. By species, measured values varied from predicted values with absolute t no more than 0.53.

Site index for aspen was estimated according to Eq. [1] from both prelog and 25 year postlog data.

Analysis

All data were aggregated to the treatment plot level for each measurement year. The yield components were examined in turn by comparing each before and after treatment. This assures that changes observed are responses to treatment and not occluded by variability in soil or genetic character. Aspen site index was evaluated directly that way:

$$S_2 = a * S_1^b * e_i \quad [5]$$

where

S₂=estimated site index in 1990

S₁=estimated site index in 1964

The null hypothesis, "aspen site index was not altered by logging with or without burning," was tested by fitting the model

$$\ln S_2 = \ln(a) + b \cdot \ln S_1 + \ln(p) \cdot FE + \ln(q) \cdot B_2 + \ln(r) \cdot B_3 + \ln(e) \quad [6]$$

where

p, q, r = treatment parameters to be estimated
FE = fire energy, joules, first burn
B₂, B₃ = dummy variables (0, 1) for repeat burns (Weisberg 1985)
e_i = deviation about the model.

The multiple null hypothesis, "prescribed burning did not alter the scale (a) nor shape (b) parameter values in the size-age relationship (model [2]), nor in the density-size relationship (model [3]) of aspen and other woody plants," was tested by fitting

$$\ln W = \ln(a) + b \cdot \ln A + k_i \cdot C_i + \ln(p) \cdot B_1 + \ln(q) \cdot B_2 + \ln(r) \cdot B_3 + \ln(e) \quad [7]$$

and

$$\ln N = \ln(a) - b \cdot \ln W + \ln(c) \cdot W + k_i \cdot C_i + \ln(p) \cdot B_1 + \ln(q) \cdot B_2 + \ln(r) \cdot B_3 + \ln(e) \quad [8]$$

where

k_i = parameter for the ith covariate, C_i.

Covariates included aspen basal area, stocking density, and site index from 1965 data, and the same (except site index) from fall 1966 data (prior to the first burn). The term, ln(c)·W, in [8] accommodates the non-linear segment of the double-logarithmic self-thinning trajectory prior to attaining maximum canopy depth at tree d.b.h. = 8.5 cm. This trajectory is linear for self-thinning stands of larger trees (Perala *et al.* 1994).

Because the stands originated in different years and therefore could have endured confounding climatic conditions, cumulative growing degree days and cumulative growing season precipitation were tried as substitutes for stand age.

Models [6]-[8] as stated express treatment effect only on the intercept (ln of the scale parameter). To include treatment effect on the shape parameter, all first-order interactions of treatment with lnA and lnW were included in the analyses.

Null hypotheses regarding burning were accepted if p, q, r, and interaction parameters = 1 i.e., ln of these = 0). The logging null hypothesis was accepted if a = 1 and b = 1 in [6]. Logarithmic models were chosen because they linearized the relationships and stabilized the variance. Models were fit by backward multiple linear regression, retaining the least significant variable only if p < 0.05. All shrub species data were included but tree species data prior to measuring at d.b.h. were excluded to avoid ambiguity induced by change in height of diameter measurement. The estimation bias inherent in logarithmic equations was relieved according to Baskerville (1972) and integrated into the a-parameters given here.

RESULTS AND DISCUSSION

General Response

Earlier analysis (Perala 1974a) revealed that the first burn did not significantly affect the shrub and hardwood component but that the most productive aspen stands arose after complete clearcutting alone. This burn inhibited aspen productivity by 28 percent, partly because aspen parent roots were injured by the intense heat sustained by the massive fuel load. Alexander (1982) later estimated the fire was nearly uncontrollable, and, indeed, did escape the fireline, burning a treatment block intended as a control. (This block was exchanged for another block intended to burn.) Productivity was further slowed because the stressed parent root system was forced to initiate another crop of suckers (Berry and Stiel 1978, Perala 1979).

The first (spring) reburn was only partially effective because forest floor fuels were sparse and matted, yielding little heat energy (Perala 1974b). Flame height attained only a few decimeters, giving a fireline intensity of only about 10 kW/m (Byram 1959; SI conversion by Rothermel and Deeming 1980). Although the fire was so gentle that it only thinned the aspen suckers, it was hot enough to kill some shrubs and hardwoods; these subsequently resprouted. By contrast, the second (autumn) reburn was highly effective (Perala 1974b). Flame heights commonly attained 0.3 to 0.6 m, giving a fireline intensity of 20 to 100 kW/m. Occasionally, flames attained 2 m height, with fireline intensities exceeding 1000 kW/m. The forest floor was consumed on 10 percent of the area. Paper birch (*Betula papyrifera* Marsh.) and quaking

aspen seeded and germinated vigorously in these patches of exposed mineral soil. Some aspen sucker-bearing roots were killed by this fire, but overall aspen, hardwoods, and shrubs sprouted in abundance comparable to the first burn.

Numerical Analyses

Aspen. Regression analyses confirmed the high utility of partitioning the productivity components of aspen (table 1) to understand burning effects. The solution of model [6] indicates that site index at the end of the study varied directly with initially measured site index and inversely with energy recorded in the first fire (table 2). Repeat burns did not further alter site index. The relation between starting and ending site values is especially strong, while the strength of

the relation to fire energy depends on the disposition of two statistical outliers (fig. 1). Without them, the relationship is unequivocal; retaining them casts some doubt. No variable measured in this study could account for these outliers.

The overall increase in apparent site index, while statistically significant, may not be real because site index estimation bias is greatest for young stands (Gevorkiantz 1956). It would be imprudent to infer overall improved site quality until these stands approach index age.

Mean tree size varied directly with parent stand basal area and site index (model [7], table 3) but parent stand attributes did not affect stockability (model [8], table 3). Neither growing degree days nor growing season precipitation

Table 1.—Equation fit statistics

Model	Cases	Adjusted R ²	Standard deviation
Aspen			
[6]	12	0.9998	0.0465
[6] ^a	10	1.0000	.0127
[7]	33	.9873	.1515
[8]	33	.9801	.0998
Hardwoods			
[7]	33	.8680	.4160
[8]	33	.7030	.2605
Shrubs			
[9]	76	.9116	.2659

^a Without statistical outliers (fig. 1).

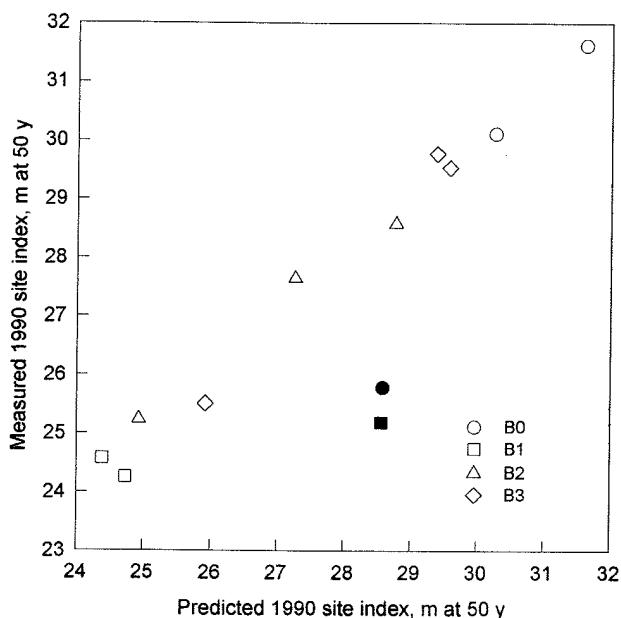


Figure 1.—The relationship between measured 1990 site index and site index predicted from eq. [6b], table 2. The shaded symbols are statistical outliers.

Table 2.—Unweighted least squares linear regressions of model [6]

Model	Dependent variable	Predictor variable	Coefficient	Student's t	Probability
[6]	lnS ₂	lnS ₁	1.107	1.52+2	0.000
		FE	-5.680-7	-2.61	.026
[6b] ^a	lnS ₂	lnS ₁	1.119	4.80+2	.000
		FE	-7.648-7	-1.18+1	.000

^a Eq. [6] data, less two statistical outliers (fig. 1).

offered improvement over stand age as the strongest predictor variable for growth. The scale and shape parameters for both size and stockability were affected by burning (table 3). For aspen size, the pattern of a-parameter values is $B3 > B0 = B1 = B2$ while the order for b-values is conversely $B3 < B0 = B1 = B2$ (table 4). These equations predict that among treatments the B3 aspens average largest at age 10 but smallest by age 25 (table 5).

Stockability of aspen at age 25 years was reduced 22 percent by the initial burn, according to the a-parameter values for model [3] (table 6).

This burn, however, did not affect the self-thinning b-value, whereas the repeat burns induced lower self-thinning rates (table 6) that enabled stocking to catch up with and even surpass the other treatments (table 7).

With only four treatments to examine, it is premature to suppose that the a- and b-values for size and stockability are in general controlled by treatment. The most that can be said is that different burning regimes will each regenerate a stand varying in initial density, stockability, and mean size. These stand attributes act and interact on each other according to models [2] and [3] to travel along unique but complementary trajectories.

Table 3.—Unweighted least squares linear regressions of models [7] and [8]

Model	Dependent variable	Predictor variable	Coefficient	Student's t	Probability
Aspen					
[7]	lnW	ln(a)	-1.714+1	-1.16+1	0.000
		lnA	3.438	4.69+1	.000
		lnG ₁ ^a	1.300	7.60	.000
		lnS ₁	1.846	3.75	.001
		B3	1.602	2.92	.007
[8]	lnN	ln(a)	9.627	2.59+2	.000
		lnW	-4.045-1	-1.37+1	.000
		W	-1.450-2	-4.73	.000
		B1	-2.543-1	-6.05	.000
		lnW*B2	4.979-2	2.28	.031
		lnW*B3	1.078-1	3.95	.001
Hardwoods					
[7]	lnW	ln(a)	-1.392+1	-7.74	.000
		lnA	3.036	1.19+1	.000
		lnN ₁ ^b	7.787-1	2.96	.006
		B1	4.062	3.82	.001
		lnA*B1	-1.302	-3.40	.002
		lnA*B3	-1.678-1	-2.09	.046
[8]	lnN	ln(a)	1.449+1	1.40+1	.000
		lnW	-2.532-1	-6.22	.000
		N ₁	-8.478-1	-5.16	.000

^a G₁ = parent stand aspen basal area, m²/ha.

^b N₁ = parent stand aspen density, trees/ha.

Table 4.—Equation parameters for size (model [2]) by vegetation class and burn treatment, calculated from model [7]

Burn treatment	Aspen parameters		Hardwood parameters	
	a	b	a	b
B0	5.434-4	3.438	1.299-4	3.036
B1	5.434-4	3.438	7.555-3	1.734
B2	5.434-4	3.438	7.555-3	1.734
B3	2.696-3	2.907	7.555-3	1.567

Table 5.—Mean aspen biomass by burning treatment and age from last initiation (model [2], using parameters in table 4)

Burn treatment	Age (yrs)			
	10	15	20	25
	----- kg -----			
B0	1.48	6.07	16.5	35.1
B1	1.48	6.07	16.5	35.1
B2	1.48	6.07	16.5	35.1
B3	2.19	7.20	16.7	31.7

Table 6.—Equation parameters for stockability (model [3]) by vegetation class and burn treatment, calculated from model [8]

Burn treatment	Parameters		
	a	b	c
Aspen			
B0	1.524+4	-4.045-1	9.856-1
B1	1.182+4	-4.045-1	9.856-1
B2	1.182+4	-3.547-1	9.856-1
B3	1.182+4	-2.967-1	9.856-1
Hardwoods			
B0	9.963+3	-2.532-1	1.000
B1	9.963+3	-2.532-1	1.000
B2	9.963+3	-2.532-1	1.000
B3	9.963+3	-2.532-1	1.000

Table 7.—Aspen stocking by burning treatment and age (model [3], using parameters from table 6, mean tree weight from table 5)

Burn treatment	Age (yrs)			
	10	15	20	25
----- trees per hectare -----				
B0	12,717	6,726	3,863	2,171
B1	9,862	5,216	2,996	1,683
B2	10,058	5,706	3,444	2,009
B3	9,078	5,927	4,020	2,675

Table 8.—Standing crop^a by vegetation class, burn treatment, and age from last initiation

Burn treatment	Age (yrs)			
	10	15	20	25
----- Mg per hectare -----				
Aspen				
B0	18.5	40.9	63.6	76.2
B1	14.6	31.7	49.3	59.1
B2	14.9	34.7	56.7	70.6
B3	19.8	42.7	67.2	84.8
Hardwoods				
B0	2.31	5.86	11.3	18.6
B1	5.11	8.70	12.7	16.9
B2	5.11	8.70	12.7	16.9
B3	3.90	6.32	8.9	11.5
Shrubs				
B0	.94	1.21	1.27	1.18
B1	1.24	1.59	1.67	1.55
B2	.96	1.23	1.29	1.20
B3	2.49	3.20	3.35	3.11
Total				
B0	21.8	48.0	76.2	96.0
B1	21.0	42.0	63.7	77.6
B2	21.0	44.6	70.7	88.7
B3	26.2	52.2	79.5	99.4

^aTree standing crop is the product of models [2]*[3] as given in tables 5 and 7. Shrub standing crop from eq. [9].

The product of table 5 x table 7 gives an estimate of aspen standing crop (table 8). The results have good precision and accuracy and are free of bias (fig. 2). The estimates indicate that the single burn reduced production over 25 years by 22 percent, whereas the repeat burns mitigated the effects of the first burn. Productivity of aspen in the autumn reburn even surpasses the control by 11 percent at age 25.

Associated hardwoods. Models [7] and [8] may not be conceptually appropriate to apply to the hardwood component that is subdominant to the aspen component. Although model fit was not as good (table 1), strong relationships nevertheless emerged (table 3). For the sake of consistency, and for lack of better models, this analysis was accepted for hardwoods as for aspen.

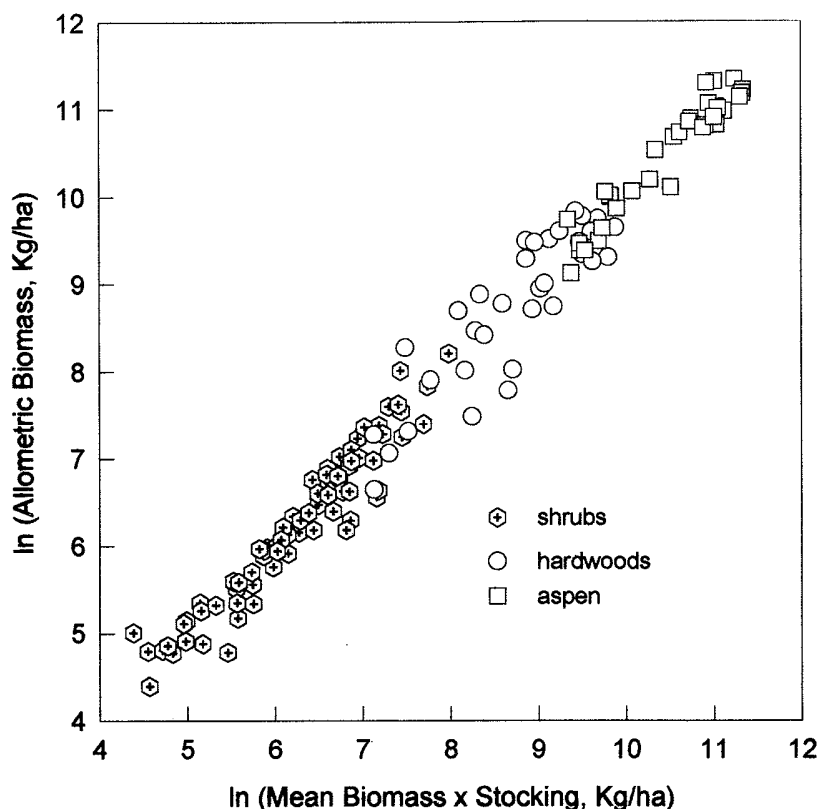


Figure 2.—The double logarithmic relationship between allometrically determined standing crop (Perala and Alban 1993) and standing crop estimated from the product of models [7] and [8] (parameters from tables 5 and 7 for trees) or from model [9] (parameters from table 9 for shrubs).

Table 9.—Unweighted least squares linear regression of model [9]

Predictor variable	Coefficient	Student's t	Probability
ln(a)	4.871	6.64	0.000
lnA	1.715	1.18+1	.000
A	-9.026-2	-5.81	.000
G ₁	7.209-2	5.53	.000
S ₁	-1.424-1	-5.08	.000
B1	2.716-1	2.28	.026
B2	-2.553-1	-2.24	.029
B3	6.991-1	6.20	.000

Both mean size and stockability of hardwoods were influenced by the number of trees in the parent stand rather than its basal area (table 3). Burning greatly increased the a-parameter value and reduced the b-parameter value for growth (table 4) but did not affect stockability or survival (table 6). The net effect was initially greater yield for burned stands but eventual smaller yields, especially after the autumn reburn (table 8).

Shrubs. Models [2] and [3] simply were inapplicable to the shrub component. A quadratic-like nonlinear model

$$W*N=a*A^b*c^A*e_i \quad [9]$$

was fit in the linear logarithmic mode along with the adjunct dummy and covariate variables (table 9) with about the same precision as the hardwood data (table 1). Shrub biomass increased with parent stand basal area but diminished with site index (table 9). Solving for mean values of these and retransforming the coefficients gives the nonlinear solutions:

Treatment	Parameter		
	a	b	c
B0	135	1.72	0.914
B1	177	1.72	.914
B2	137	1.72	.914
B3	357	1.72	.914

Thus, the b- and c-values indicate an asymptotic approach to a maximum shrub biomass at about 20 years followed by decline (table 8). The a-value shows that one burn increased shrub mass by 31 percent. Although the autumn reburn further doubled shrub productivity, the spring reburn lowered shrub productivity to the unburned level.

CONCLUSIONS

The effect of fire on stand component and total yield after 25 years is considerable. The yield of once-burned aspen remains diminished by 22 percent at age 25, compared to the 28 percent deficiency first reported (Perala 1974). Reduced yield appears to be attributable to inferior stockability induced by fire energy and a weakened physiological state. In contrast, the 9 percent lesser yield of hardwoods at age 25 is the result of slower growth. Repeat burns favored the aspen component by enhancing

stockability while they diminished the hardwood component by further slowing growth, at least in the autumn reburn. It is remarkable that aspen productivity after the autumn reburn recovered to 111 percent of the unburned aspen when fire history has been considered detrimental to aspen site index (Van Cleve 1973). Stoeckeler (1960) observed a 3-meter reduction in aspen site index in stands enduring repeat burns. The effects he describes, however, appear to be related to condition of surviving trees (fire scars, root injury) and his observations may not apply to stands that are completely killed (or nearly so). Weber (1990) also noted that gentle surface fires only weaken aspen, causing eventual mortality with little suckering (see also Quintillo *et al.* 1991) while stands completely killed by intense fire sucker profusely.

How aspen stands might benefit in some cases from fire is not clear. Fire intensity, clonal variability, site character, and phenological stage are a few variables that might confound response. Reich *et al.* (1990) found elevated foliar N, P, and K in *Acer rubrum* L., *Prunus serotina* Ehrh., and *Quercus ellipsoidalis* E. J. Hill following a gentle surface fire in the understory. The latter two, but not *Acer rubrum*, responded with increased photosynthetic rates. Would aspen respond likewise?

In summary, aspen yield may be less after burning slash left by clearcutting, but this study shows that repeated burning may ameliorate growth. Ultimately, aspen yield is determined by conditions that control growth, stockability, and site index. Site index was diminished by the initial burn and did not recover regardless of ensuing history. The response to subsequent burning suggests mitigation of the factors controlling stockability, thought to be related to the water balance (Perala *et al.* 1994). The responsible mechanism is not apparent from these data. Tree growth may follow different trajectories accompanied by more-or-less complementary survival trajectories. Thus stands may eventually converge on the same yield, distributed over different numbers of trees.

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